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Variable Stiffness Robotic Hand for Stable Grasp and Flexible Handling

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ABSTRACT Robotic grasping is a challenging area in the field of robotics. When interacting with an object, the dynamic properties of the object will play an important role where a gripper (as a system), which has been shown to be stable as per appropriate stability criteria, can become unstable when coupled to an object. However, including a sufficiently compliant element within the actuation system of the robotic hand can increase the stability of the grasp in the presence of uncertainties. This paper deals with an innovative robotic variable stiffness hand design, VSH₁, for industrial applications. The main objective of this work is to realise an affordable, as well as durable, adaptable, and compliant gripper for industrial environments with a larger interval of stiffness variability than similar existing systems. The driving system for the proposed hand consists of two servo motors and one linear spring arranged in a relatively simple fashion. Having just a single spring in the actuation system helps us to achieve a very small hysteresis band and represents a means by which to rapidly control the stiffness. We prove, both mathematically and experimentally, that the proposed model is characterised by a broad range of stiffness. To control the grasp, a first-order sliding mode controller (SMC) is designed and presented. The experimental results provided will show how, despite the relatively simple implementation of our first prototype, the hand performs extremely well in terms of both stiffness variability and force controllability.

INDEX TERMS Variable stiffness hand, stable grasp, force control, sliding mode control (SMC).

I. INTRODUCTION

The uncertainty associated with miscalculated grasp models and/or objects with unknown mechanical parameters create difficulties in performing a stable grasp. Traditional approaches to eliminating this problem involve robotic hands that are expensive, delicate, complex and difficult to control. However, including a sufficiently compliant element within the actuation system of the robotic hand can provide an alternative solution to this challenge. Integrating such a passive component into a robotic system will increase the stability of the grasp in the presence of uncertainties. This is also true in the human hand, as it has been demonstrated that the passive nonlinear dynamics of the joints in the human hand play a vital role in providing a stable grasp [1]-[7].

The passive behaviour of the human body, and more specifically the human hand, is the result of a combination of both parallel and series compliance. This form of behaviour at the metacarpophalangeal joints is largely due to the elasticity of the capsular ligament of the joints and muscle-tendon units [6], where the latter contributes to the stiffness of the joints by generating force when the muscle or tendon is under some form of tension [7]. Most of the existing research on robotic systems with variable stiffness/compliance take inspiration from the human body, mainly because of the aim of developing artificial limbs [8]-[15]. However, certain fundamental concepts and ideas that arise from this line of research can be exploited in order to create a new generation of industrial robots, and more specifically industrial grippers/hands, which feature controllable stiffness for demanding industrial applications requiring flexibility in grasping tasks. In-depth discussions about human hand grasping and human body impedance modulation can be found in [1]-[8], to mention but a few. Following these and other similar studies, a plethora of variable stiffness/compliance designs have been proposed for robotic systems over the last decade [9]-[18].

One of the earliest attempts to produce compliant actuators was accomplished by Pratt et al. [14]. They suggested an elastic element should be placed between the conventional rigid actuators and external loads. They also developed one of the earliest impedance control methods for their serial elastic actuator. They showed some of the benefits of using such actuators, which include shock resistance, smaller sensible inertia, more precise and robust force control, safer interaction with the environment and energy storage properties. This
actuator was used as the actuation system in the arms of the MIT humanoid robot “COG” [15], [16]. A compact rotational series elastic actuator was introduced in [18], [19], for use in multi-DOF small-scale humanoid robots. The design consisted of six identical linear mechanical springs and a conventional DC motor. There were three rigid spoke elements connecting the central bearing (output shaft) of the actuator to the springs. This connection was used to transfer the elasticity of the springs to the main shaft and hence provide compliance at the shaft.

The Compact Rotary Series Elastic Actuator (cRSEA) was suggested by Kong et al. [20], [21] to be used in human assistive limbs. They used a combination of a torsion spring and a chain of worm and spur gears in this design to reduce size and achieve precise torque control. To control the output torque of the actuator (assistive torque) they used real-time feedback of the joint angle and environmental contact force. The gear-spring mechanism in their system isolated the motor from the environment and hence could potentially be used as a shock absorber.

Tonietti et al. proposed a variable stiffness actuator designed for use in robotic systems as well as any mechanical devices which require some form of physical interaction with their surrounding environment [22], [23]. The actuator consisted of two DC motors. The shaft of each DC motor was connected to a pulley. A timing belt connected the two DC motors and their associated pulleys to the output shaft. Three compression springs were used so as to create tension on the belt in their rest positions. In order to control the position of the output shaft, both DC motors were rotated in the same direction (and at the same speed), whereas the (remove the) rotation of the motors in the opposite direction changed the apparent stiffness of the output shaft.

A new compliant joint actuated by an antagonistically-twisted round-belt actuator was proposed by Inoue et al. [24], to be used in robotic applications. The design comprised two DC motors, one pulley, and a link connected to this pulley. Two twisting elastic and flexible round-belts connected the pulley to the shaft of the DC motors in an antagonistic setup. The contraction generated by twisting the belts was used to create a moment, and consequently rotational motion on the pulley [24].

In order to achieve a compliant leg for bipedal robots, a mechanically adjustable compliance and controllable equilibrium position actuator (MACCEPA1) was suggested in [25]-[27]. The MACCEPA consisted of three links and one common revolute joint (knee joint), where the links pivoted around the knee joint. There was also a lever link connected to the knee. A linear tension spring was attached to the lever link and a string connected this spring to the lower link. The angle between the upper link and lever link could be changed by an electrical motor connected to the lever link. When the angle between lever link and the lower link was not zero, any elongation of the spring would generate a resistive torque, trying to line up the lower link with the lever link. When this angle was zero (the equilibrium position) the spring would not apply any resistive torque to the lower link. To generate an elongation on the spring and consequently a resistive torque, they used an electro-motor. This electro-motor was used to pull on the cable connected to the spring, which resulted in the pre-tensioning of the latter. This pre-tension changed the resistive torque for a given angle, consequently changing the apparent stiffness of the system.

Another relatively similar approach to the variable stiffness actuator was introduced in [28], [29]. The model consisted of a linear compression spring connected to a low-friction roller on one side and a linear actuation mechanism on the other. Similar to MACCEPA, the role of the linear actuation system in this model was to generate a pre-tension on the linear spring by compressing it. The output link of the system was connected to a concave nonlinear cam and a revolute joint was used to connect this cam to the main chassis. The roller was able to move inside the concave surface of the cam with a very low friction, with the associated motion used to generate the apparent stiffness of the output link. To change the apparent stiffness of the system, the linear motor was used to change the length of the spring and, consequently, the stiffness of the joint. The apparent stiffness of the system was a nonlinear function of the stiffness constant of the spring, the cam transmission ratio, and the offset of the output link. As mentioned, the design used a single actuator to change the output stiffness; however, the system was unable to control the output position (position of the output link).

A simplified model of the pulley-belt driven variable stiffness actuator was suggested by Grebenstein et al. [30], [31], and has been used as the actuation system in the DLR hand [32]. The model consisted of a DC motor, a pulley-tendon system and a slider-spring mechanism. The slider-spring mechanism was made of a linear compression spring, which was used to push the tendon in its rest position, forming the tendon into a triangle. To achieve an independent position and stiffness controllability in each joint, they used a pair of the mechanism detailed, in an antagonistic manner for each joint.

As explained before, the inherent passive properties of the human hand, in both serial and parallel combinations, play an important role in grasp stabilisation [33]-[37]. Various studies into the grasp of the human hand have shown that as a preliminary response, to achieve a robust grasp, humans tighten their fingers by co-contracting antagonistic muscles and consequently increase the stiffness of the fingers just before perceiving impact [33].

Kajikawa et al. designed a four finger, twelve joint variable stiffness robotic hand for human care service tasks. To reduce the number of the actuators in the hand, they suggested a linkage mechanism which coupled the distal and proximal interphalangeal joints and actuated these two joints via a single motor. To achieve compliance in the joints, they used silicon made from deformable cushions called SRC \textsubscript{trans}. An expandable cushion, SRC \textsubscript{stiff}, has been used to compress the
SRC\text{max} and consequently change the stiffness of the fingers. They used air pressure to inflate the SRC\text{eff} [38].

A three-joint variable stiffness robotic finger was introduced by Yang et al. [39]. Their design consisted of a soft pneumatic muscle and three pin heaters which were embedded in a shape memory polymer (SMP). The finger could bend by selectively heating the SMPs and due to internal air pressure of the pneumatic muscle. Additionally, the finger exhibited variable stiffness at different SMP’s temperatures.

Yap et al. introduced a soft wearable exoskeleton glove for assistive and rehabilitation applications. They used embedded pneumatic actuators to actuate the exoskeleton. They showed that the stiffness of the fingers could be changed in different locations; however, this stiffness was not controllable [40].

Inspired by the human hand’s tendon routings, and with the aim of improving the grasp stability and dexterity in manipulation tasks, a parallel compliant joint has been suggested for robotic fingers in [42]. The design consisted of a rectangular-shaped compliant material which was fixed between a pair of pulleys. The pulleys were fixed to the rotating shaft of the joints in such a way as to allow them to rotate with the fingers about the fingers’ revolute joints. In order to fix the compliant part, they used two fixed pairs of pins. To prevent the compliant part from undergoing any undesired displacement, they used two clamps at the top of the pins. The rotation of the joint induced a tension on the compliant material, and consequently the compliant material created a passive torque due to its intrinsic properties.

To achieve appropriate mechanical impedance properties for the wide range of joint angles inherent to the human hand, they suggested a design optimisation method. They used this method to optimise the design variables (radius of joints, pulley and pins, the distance between centre of pulleys and joint and the thickness of the compliant component). Using an open-loop motion control to execute certain grasps, they experimentally proved that adding a parallel compliant component to the finger joints could improve the quality of the grasp. To emphasise the role of the suggested parallel stiffness, they first demonstrated that the feedback delay can destabilise the grasping task. Afterwards they concluded mathematically, as well as experimentally, that adding parallel compliance part to the gripper’s joint can reduce the sensitivity of the gripper to this delay and consequently increase the stability of the grasp [36]-[37].

Using polymer-based Shape Deposition Manufacturing (SDM), Dollar et al. designed and fabricated an under-actuated, adaptive and compliant grasper [41], [43]-[45]. To increase the friction and prevent undesired slippage, the grasp side of each link contained a soft finger pad. A compliant joint flexure with a stiffness range between 0.0421 and 0.224 Nm/rad was used in the proximal and distal joints to connect the finger links. An embedded Hall Effect sensor in each joint was used to provide feedback regarding joint angle. A pre-stretched, nylon-coated, stainless-steel cable anchored into the distal link was used to transfer the actuation force from the actuator to the fingers and hence provide the motion. In zero actuation mode, the tendons, which were parallel with the flexible joints, remained slack, and hence the fingers remained in their maximum compliant mode. In actuation mode, however, the inelastic tendons reduced the flexibility of the fingers (increasing the fingers’ stiffness), consequently increasing the accuracy of the grasp. The stiffness constant of the joints was 0.19 Nm.deg for both proximal and distal joints, as based on the optimisation studies they developed to create a functional grasper. They showed that this stiffness enables the grasping of the widest range of object sizes with the greatest amount of uncertainty in object position [45]. They also showed that the uncertainty of the grasping tasks can be satisfactorily accommodated by having optimal compliance and adaptability in the mechanical design of the hand. The experimental results provided demonstrated the robustness of the SDM hand in grasping objects in the presence of large positional errors [50].

Pettersson et al. proposed a gripper mechanism that utilised the magnetorheological (MR) fluid in its variable impedance actuation mechanism. The gripper was designed for pick and place tasks in natural food product companies where the objects have different shapes and can be easily squashed. Reducing the risk of bruising through variable impedance gripping was the main advantage of the design, as claimed in [46].

Maekawa et al. developed a three-fingered robot hand with a new method of controlling stiffness. Briefly, the hand was formed from three fingers, each of which included three joints. A tendon-sheath actuated by D.C. servo motor was the driver mechanism for each joint. An embedded potentiometer and a new tension differential-type torque sensor were used to provide torque feedback from each joint. They proposed a stiffness control scheme to control the apparent stiffness of the hand. By using the joints’ positions and torque feedback, the controller was controlling both the position and stiffness of the joints and, consequently, controlling the grasp impedance. Finally, they validated the proposed mechanism and designed a position-stiffness control method by conducting various grasping experiments [47].

Inspired by human hand, Lau et al. designed a low cost, variable stiffness anthropomorphic robotic hand using pneumatic artificial muscles. Their proposed anthropomorphic design consisted of 16 DOFs in which 14 pneumatic air muscles were used to actuate the tendon-driven fingers. They used an open-loop control scheme to control the fingers’ positions and stiffnesses. The hand was able to perform some basic grasps [48].

RAMA-1 was a highly dexterous 48 DOF robotic hand designed by Rasakatla et al. [49]. The robot consisted of joints which were based on magnetic sliding and spherical spheres and used tendons to actuate the fingers. It provided more degrees of freedom than the human hand. The new six DOF thumb in this hand had a greater range of motion than the ordinary thumb and improved the overall dexterity and
manipulability of the hand. They tried to simplify the process and control task for robust grasping. They demonstrated through experiment that using an optimised passive compliant joint and adaptive coupling in the hand increases the adaptability of the large positioning errors that can occur in unstructured grasping tasks.

To control the mechanical impedance of the fingers in a flexible joint hand, two new control designs were proposed in [51]. The target impedance in the authors’ grasp model was based on the desired stiffness and damping, though they neglected the inertia term of the impedance in their model. The two suggested cascade controllers consisted of one inner torque-feedback loop, and an outer impedance control loop. They used a physical interpretation of the rotor inertia to estimate the torque in the inner loop of the controllers. They then designed two different outer impedance controllers. The first controller used a combination of the motor shaft’s position and the system’s stiffness and damping term to control the impedance of the grasp, whereas in the second controller these parameters were merged such that under steady-state conditions the desired equilibrium position could be satisfied. They also demonstrated that both controllers could be adapted to the visco-elastic properties of the joints. They experimentally verified the concept of the controllers using a DLR lightweight hand.

To conclude, any uncertainty inherent to the grasp model can easily destabilise the interaction port (fingers-object contact point) of the grasp when controlled by a conventional fixed gain control method. Fig. 1 illustrates how the mathematical properties of the object (stiffness, \(K_b\)) can move the system’s poles to the right side of the root locus plot, which leads to the grasp being destabilised.

One of the traditional approaches to eliminating this undesired destabilising effect is to include a passive elastic element between the fingers and actuator. This passive element can increase the stability of the system in the presence of such uncertainties. A range of variable stiffness/compliance designs have been reviewed and discussed in this section.

Although the variable stiffness mechanisms reviewed can still, to some extent, increase the stability of the system when interacting with the environment, their application in real-world industrial scenarios is still somewhat lacking. As far as grippers/hands are concerned, the complexity of the design, small operational force and stiffness range, weight, durability and cost issues are amongst the various reasons that might cause industry to insist on the continued use of traditional stiff mechanisms.

This paper deals with an innovative robotic variable stiffness hand design, VSH\(_1\), for industrial applications. The proposed passive, adjustable compliance, serial elastic actuation system introduced in this paper is suitable for industrial applications which greatly reduces the limitation of the maximum achievable stiffness. Our design consists of only two servo motors, the combined motion of which are used to drive the fingers and change the compliance of the joints. Non-stretchable tendon is used to transfer the actuating force to the fingers. The design provides a fast response solution by which to control the grip impedance; simplicity of design, small hysteresis band and affordability, as well as durability, are amongst their advantages. The overall architecture of the concept is based on the principle that a simple mechanism provides inherent robustness and reliability and, therefore, is able to withstand the severe working conditions inherent to the long and repetitive tasks typical of production lines.

The remainder of this paper is organised as follows: Section II. A provides an overview of our proposed variable stiffness hand, called VSH\(_1\), followed in sub-section B, by a discussion of the stiffness model in both stiff and compliant status and the mathematical modelling of the hand’s apparent stiffness and its associated force-displacement function. In Section III a report on the experimental results on the hand’s performance for different stiffness values will be given.

Sections IV provides a discussion on the first-order sliding mode control method we designed and validated (in Section V) to control the grip force. The paper will end with final discussion and conclusions in Section VI. Note that the terms “stiffness” and “compliance” and related adjectives (“compliant”, “stiff”) are used herein to indistinctly characterise, as opposing terms, the non-rigid behaviour of the gripper actuation system.

II. VARIABLE STIFFNESS HAND (VSH\(_1\))

In this section we introduce the design of a novel variable stiffness hand (VSH\(_1\)) for industrial robotic manipulators, which can be used for stable grasps (as discussed before) with unknown objects to be grasped, as well as to control the applied grip force in the absence of any accurate force sensor.

A. DESIGN EXPLANATION

Similar to the majority of variable stiffness mechanisms that are referenced in this paper, our design consists of two rotational electric actuators. The actuators are two identical 7 Nm servo motors whose mechanical and electrical details are
reported in Table I. A side view of VSH₁ with two different versions of fingers (two finger, two joints (left) and three finger, six joints (right) can be seen in Fig. 2. As shown in this figure, both versions of the VSH₁ use an identical actuation system to actuate the fingers.

![A side view of VSH₁ with two different versions of fingers](image)

FIGURE 2. The VSH₁ introduced in this paper with two different versions of fingers (two finger, two joints (left) and three finger, six joints (right)).

The two servo motors also can be seen in this figure. One of the servos, M₁, provides rotational motion θ, whilst the second is used to produce a linear displacement, ΔB, along the wrist axis of the gripper. Fig. 3 depicts these servos and their corresponding motions. As can be seen from this figure, a tendon-pulley-slider arrangement is used to transform the rotational motion of M₂ to achieve linear displacement. We used this linear motion to move the slider along the wrist axis of VSH₁ as shown in this figure. Motor M₁ is mounted on this slider and follows the slider’s movements. As shown in the figure, there is a linear compression spring connected to the shaft of M₁ through a rigid rod. Fig. 3 also depicts a pair of pins in the centre of the shaft. We use these pins to hold the rod and spring. The rod slides through the pins and across the shaft’s axis. The linear compression spring is placed around the rod, also as shown.

![A spring holder pin in the bottom of the rod holds the spring in place](image)

A spring holder pin in the bottom of the rod holds the spring in place. To transfer the driving force from the actuator to the fingers, a tendon establishes the connection between the rod and fingers.

![Schematic representation of our VSH₁](image)

FIGURE 3. (a) A CAD model and (b) a schematic representation of our VSH₁. Our design consists of two rotational electric actuators M₁ and M₂. One of the servos, M₁, provides rotational motion θ, whilst the second is used to produce a linear displacement, ΔB, along the wrist axis of the gripper.

The subsequent tendon-rod-spring configuration generates a compliance behaviour for the hand which will be explained in the upcoming sections. Any external force on the hand’s fingers will generate a tensile force which will be transferred to the rod-spring system via the tendon. The force transferred to the rod will pull it, and consequently compress the spring where, as will be explained in the following section, the magnitude of this compression is a function of the force and θ.

**B. WORKING PRINCIPLE AND MODELLING**

In this section, we will explain the working principle of our variable stiffness mechanism. To do so, as depicted in Fig. 4, we use two coordinate frames: (a) the reference coordinate frame OXYZ, and (b) the shaft coordinate frame o'x'y'z' which is parallel to the reference coordinate frame. We assume that the shaft coordinate frame is fixed to the shaft of M₁ in

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such a way that $o'$ is in this shaft’s geometric centre, as shown in the Figure. From this figure, the rotational motion of $M_1$ is around $x'$ axis of $o'x'y'z'$ and the linear motion of $M_2$ is along the $Y$ direction of $OXYZ$. The combination of motions of $M_1$ and $M_2$ provides the ability to independently control the stiffness and position of the fingers, as will be explained below.

In Fig. 4, the purple line represents the tendon that establishes the connection between the rod and fingers. As already mentioned, this tendon is used to transfer the driving force from the variable stiffness mechanism to the fingers. The apparent stiffness of the fingers, $\delta_o$, is dependent on the angle between the rod and tendon, and this stiffness changes according to this angle. We use $M_1$ to change this angle, and hence control the stiffness of the fingers.

Now let us remove the hand from its actuation system and assume that the tendon is pulled by an external force, $F$, as shown in Fig. 5. Hence, we can write:

$$|F \cos \alpha_1| = KZ_s'$$

(1)

Where $\alpha_1$, as shown in the figure, is the angle between the rod and tendon, $Z_s'$ is the amount of compression in the spring due to the force $F$, and $K$ denotes the spring constant. From (1) we can write:

$$Z_s' = \frac{|F \cos \alpha_1|}{K}$$

(2)

We assume that the rotation of the servo motor $M_1$ is bounded as below:

$$0 \leq \theta \leq \frac{\pi}{2}$$

(3)

From Fig. 5, (3) and (2) we can write:

$$\pi \leq \alpha_1 \leq \frac{\pi}{2}$$

(4)

$$0 \leq |Z_s'| \leq \frac{|F|}{K}$$

Equation (4) illustrates the minimum and maximum compression of the spring (which is equal to the displacement of the rod) due to the external force $F$ on the tendon. In this equation, zero compression corresponds to $|\alpha_1| = \pi/2$; assuming the motors are non-back driveable, the tendon is inelastic, and the shear deformation of the rod is negligible, we can thus write:

$$|\alpha_1| \rightarrow \frac{\pi}{2} \quad \delta_o \rightarrow \infty$$

(5)

and

$$\min \delta_o = K|\alpha_1=\pi$$

(6)

where $\delta_o$ is the output stiffness of the variable stiffness mechanism. Fig. 6 shows the hand in both its open and closed states with minimum and maximum stiffness. In this figure, the red hands correspond to the stiff fingers with the maximum (ideally infinite) stiffness, whilst the blue ones correspond to the fingers with minimum stiffness, which is equal to the stiffness of the spring, $K$.

As shown in this figure and from (5) and (6), in both states related to the hand with maximum stiffness, the angles between the tendon and rod are a right angle and for the blue hands, which correspond to the minimum stiffness, the tendon lies along the rod and spring ($\alpha_1 = \pi$).

In order to derive the stiffness function of the hand, let us assume that the displacement of the tendon due to the above-
mentioned external force is equal to \( d \). Fig. 7 depicts this force and its associated displacement. For clarity, we have not shown the hand in this figure. \( D_0 \) represents the distance between rod’s end point (tendon-rod fixing point) and the wrist of the hand. \( Z' \) is the length between rod’s end point and the centre of the shaft of \( M_1 \) before applying the force. \( D_1 \) is the distance between rod’s end point and the wrist after applying the force. \( Z' \) shows the displacement of the rod’s end point due to compression of the spring after applying the force.

Using the law of sines, we can write:

\[
\sin \theta = \frac{D_0 \sin \alpha_0}{B} \tag{7}
\]

where \( \theta \) is the angle between the rod and \( Y \)-axis, \( B \) is the distance between the centre of the shaft of \( M_1 \) and wrist point of the hand, and \( \alpha_0 \) is the angle between the tendon and rod in their initial positions. After applying the force, and by using (7), we can write:

\[
\frac{\sin \alpha_1}{B} = \frac{D_0 \sin \alpha_0}{BD_1} \tag{8}
\]

Assuming that the tendon is perfectly inelastic, we can write:

\[
D_1 = D_0 - d \tag{9}
\]

Using (8) and (9) we have:

\[
\alpha_1 = \sin^{-1} \frac{D_0 \sin \alpha_0}{(D_0 - d)} \tag{10}
\]

And by again using the sine law and (9), we can get:

\[
\frac{\sin(\beta_0 + \beta_1)}{z' + z'_s} = \frac{\sin \theta}{D_0 - d} \tag{11}
\]

where \( \beta_0 \) and \( \beta_1 \) are shown in Fig. 7. By simple calculation, we get:

\[
\beta_1 = \alpha_0 - \alpha_1 \tag{12}
\]

substituting (12) into (11) we obtain:

\[
z'_s = \frac{(D_0 - d) \sin(\beta_0 + \alpha_0 - \alpha_1)}{\sin \theta} - z' \tag{13}
\]

Finally, from (10) and (13) we can conclude:

\[
z'_s = \frac{(D_0 - d) \sin(\beta_0 + \alpha_0 - \sin^{-1}\left(\frac{D_0 \sin \alpha_0}{(D_0 - d)}\right))}{\sin \theta} - z' \tag{14}
\]

Also, from Fig. 6 we have:

\[
F' = -F \cos \alpha_1 = Kz'_s \tag{15}
\]

where \( F' \) is the decomposed element of \( F \) along the rod axis. Adding (10) and (14) to (15) we can write:

\[
F = K(d - D_0) \sin(\beta_0 + \alpha_0 - \sin^{-1}\left(\frac{D_0 \sin \alpha_0}{(D_0 - d)}\right)) + z' \sin \theta
\]

where:

\[
\begin{aligned}
D_0 &= \sqrt{B^2 + z'^2 - 2Bz' \cos \theta} \\
\beta_0 &= \sin^{-1}\left(Z'\left(\frac{\sin \theta}{D_0}\right)\right)
\end{aligned} \tag{17}
\]

Equation (16) and (17) formalise the relationship between the applied force \( F \) and \( d \) for different \( \theta \), which entails the nonlinearity of the output stiffness \( \delta_0 \). The set of curves in Fig. 8 shows how \( d \) changes when \( F \) varies over a discrete range of \( \theta \) (from 0 to 40°) in two different views. In these figures, the lowest line shows the stiffness of the fingers when \( \theta = 0 \) (\( \alpha_1 = \pi \)). As expected, due to the linear spring used in our actuator, there is a plateau in the force-displacement relationship for this angle. The slope of this line is equal to the stiffness of the integrated spring, \( K \). From this figure, and entirely as expected, the slopes of the curves increase with increasing \( \theta \) as the highest line, the red curve, is associated to the greatest angle \( \theta = 40^\circ \).

In more generic terms, the stiffness of a grasp can be modelled by a relationship between the applied force and the displacement due to this force [52]:

\[
\delta_0^\theta = \frac{\partial F}{\partial d} \tag{18}
\]

where the term \( \delta_0^\theta \) highlights the dependence of the grasp’s stiffness on the angle \( \theta \). As shown in Fig. 8, this stiffness increases with increasing \( \theta \). It is worth noting that from (16) and (17), the fingers’ stiffness, \( \delta_o \), is also dependent on the stiffness of the spring \( K \) and the variable \( B \). Fig. 9 shows the effect of different values of \( K \) and \( B \) on the output stiffness of
the hand. As can be noted from this figure, the output stiffness of the hand (fingers’ stiffness) increases with increasing $K$ and/or $B$.

III. EXPERIMENTAL RESULTS AND VALIDATION

To validate the concept of the variable stiffness hand, we fabricated our tendon-driven hand prototype, VSH1, (Fig. 10), as characterised by three fingers and six joints (two joints per finger). This figure shows the hand and its ability to grasp objects of different stiffnesses, shapes and weights. In Fig. 11, we report actual measurements of the displacement $d$ for different values of applied force in the presence of different rotations of $M_1$ ($\theta = 5^\circ, 10^\circ, 15^\circ, 20^\circ, 25^\circ, 30^\circ, 40^\circ$ and $55^\circ$). To collect these data, we used a spring with a stiffness constant of 0.3 N/mm in VSM1. To collect the experimental results, we removed the hand from the actuation system and, by hanging different weights on the tendon, we measured the associated elongation of the tendon, $d$.

To test the capability of the fingers to follow a desired trajectory in the presence of different stiffnesses, a trajectory tracking experiment was performed using a Sin(5$t$) motion input applied to $M_2$. Fig. 12 depicts a schematic model of the hardware setup of this experiment. The output of this experiment was the rotation of the fingers (left finger in the figure) about the fingers’ joint. This rotation has been measured by a rotational encoder mounted on the joint of the left finger. Fig. 12, also depicts a fire brick placed between the fingers. This brick was used to stop the fingers in their movement at a certain position. A FSR (Force Sensitive Resistor) sensor was mounted on this brick to measure the grip force. The main technical specifications of the sensor are reported in Table II. Fig. 13 shows the experimental results collected from this experiment. To perform this test, we set the angle $\theta$ to $0^\circ$, $10^\circ$, $20^\circ$ and $30^\circ$ where the subplots a, b, c and d show the finger trajectories associated to these angles, respectively.
The green dashed lines in this figure depict the desired input trajectory (applied to M2), whereas the solid blue lines show the actual motion of the fingers measured by the encoder. As shown in this figure, the increased stiffness of the fingers acts to stabilise the system where the finger with higher stiffness ($\theta = 30^\circ$) follows the sinusoidal trajectory with a reduced error.

Fig. 13.e depicts the grip force applied by the hand to the firebrick, as measured through the FSR sensor. The figure shows that the applied force increases by increasing the value of $\theta$ and, consequently, the stiffness. The smallest force (the black dashed line in the figure) measured for the test was for the smallest $\theta$, which corresponds to the smallest stiffness. As this figure shows, the grip force was increased with increasing $\theta$. This was expected, as larger $\theta$ corresponds to a greater stiffness of the finger, so for a given displacement, the larger stiffness must generate the greater force.

Fig. 14 shows the stiffness hysteresis curves for different values of the rod angle, $\theta$, obtained by gradually applying an external force and measuring the associated displacement, $d$ and then gradually removing this force. To perform this experiment, we used a spring with a spring constant of 0.55 N/mm. Clearly this hysteresis could be narrowed through a better design aimed at reducing friction and damping in the mechanical couplings.
IV. FORCE CONTROL FOR THE VSH

In this section we discuss the sliding mode force control architecture we designed to control the grip force in our variable stiffness hand. A schematic model of the hand is illustrated in Figure 16. For clarity, the mechanical connections for only one finger and the actuation system are shown in this figure. The two masses, \( M_F \) and \( M_b \), describe the mass of the finger and rotor mass of the second servo motor, \( M_2 \), respectively. The spring \( K_e \) in this figure is used to model the output nonlinear (variable) stiffness term where \( K_e = \delta \omega \). \( K_b \) is used to model the stiffness of the grasped object. The dampers, \( B \) and \( B_b \), are used to model the friction between the fingers and palm, and the friction between the rotor and stator of the DC motor (the friction of the shaft bearings and friction between the commutator and brushes of \( M_2 \)), respectively. For simplicity, the friction between the rod and pins is neglected. Finally, the stiffness and damping terms, \( K_f \) and \( B_f \), are used to model the stiffness and frictional losses of the sliding system, respectively. As shown in the figure, the rotor is driven by the motor magnetic field force, \( F_m \).

\[
\begin{align*}
\dot{\theta}_f + B \dot{\theta}_f + K_f \theta_f + F_{du} &= k_T \left( \frac{E_{M2} - V_{CEMF}}{R_{armature}} \right) \quad \text{(19)} \\
&= k_T I_m = AT_m
\end{align*}
\]

where \( I \) is the equivalent moment of inertia for the fingers, sliding system and the motor armature. We used the damping term \( B \) in order to model all the frictional losses (the fingers’ joint, \( B \), rotor-stator ball bearings, \( B_b \), and the friction between the tendon and pulley and friction of the sliding mechanism, \( B_j \)). \( \ddot{R} \) represents the system’s stiffness and \( F_{du} \) is the disturbance-uncertainty term which includes the environmental disturbance force acting upon the fingers as well as any unmodelled parameters of the system. \( k_T, E_{M2}, V_{CEMF} \) and \( R_{armature} \) are the DC motor’s torque constant, operating voltage, counter-electromotive force (CEMF) and terminal resistance (ohms), respectively. \( I_m \) and \( T_m \) are the DC motor’s operating current (the current through the motor’s windings) and the motor’s output torque, respectively, and \( A \) is a constant. For the counter-electromotive of the DC motor we can write:

\[
V_{CEMF} = K_e \dot{\theta}_f \quad \text{(20)}
\]

where \( K_e \) is counter-electromotive force constant of the motor. Using (20) we can rewrite (19) as per below:

\[
I \dot{\theta}_f + B \dot{\theta}_f + K \theta_f + F_{du} = k_T R_{armature} E_{M2} \quad \text{(21)}
\]

where:

\[
B' = B + k_T \frac{K_e}{R_{armature}} \\
k_T = k_T / R_{armature} \quad \text{(22)}
\]

From (21), the state space model of the system can be written as per below:

\[
\begin{align*}
\dot{X}_1 &= \theta_f \\
\dot{X}_2 &= \dot{\theta}_f \\
\ddot{X}_2 &= \ddot{\theta}_f = A_{1,2,3,4}(X_1, X_2) + \ddot{F}_d
\end{align*}
\]

where \( X_1 \) and \( X_2 \) are the state variables which, as shown in this equation, are equal to the fingers’ rotational angle and velocity (\( \theta_f, \dot{\theta}_f \), respectively. \( A_{1,2,3,4} \) in this equation is a function of state variables and contains the \( I, B, K \) and \( F_{du} \) terms, whereas \( \ddot{F}_d \) is the quotient of \( I \) and \( K \).

It is worth noting that an accurate model of the grasp is hard to determine for several reasons. For instance, let us assume the grasp task in Fig. 16. As shown in this figure, the gripper should grasp an object with the stiffness of \( K_b \). Before the fingers touch the object, the stiffness of the system (\( \ddot{R} \) in (21)) has no effect on the grasp model and hence it is negligible. However, as soon as the fingers start touching the object, the stiffness of the system and the stiffness of the object need to be considered in the grasp model. Unfortunately, there is no way to calculate the stiffness of the unknown objects to be grasped; this makes the grasp model inaccurate. Note that the uncertainty in the object’s stiffness is not the only uncertainty in the grasp. In the design of any control system, and more specifically grasp control, there are always mismatches between the actual system and its dynamical model. These mismatches arise for various reasons such as external disturbances, linearization of nonlinear parameters, neglected and/or unmeasurable parameters (such as friction). In the presence of such uncertainties during grasping tasks and due to unknown external disturbances, utilising any ordinary control methods will be difficult if not impossible. Robust control methods, and more specifically sliding mode control,
however, represent an alternative solution to overcoming such difficulties [53]-[55].

From Fig. 16 it may be noted that, apart from the stabilisation effect of the integrated compliant element, this compliant element can be used to control the grip force applied as \( F_{\text{grip}} = K_0 d L \). In order to control this grip force, we designed a sliding mode-based force control method that is explained in the remainder of this section. To design a sliding mode control, we first need to design a sliding variable, \( \sigma(e, \dot{e}, \ddot{e}, \ldots) \). Let us assume an error function as below:

\[
e(t) = F_d - F_g
\]

where \( F_d \) is the desired grip force, whereas \( F_g \) is the measured force, the magnitude of which is acquired through a force sensor. The next step in designing a sliding mode control is defining a sliding variable. The sliding variable for the above error state is given by:

\[
\sigma_{(e, \dot{e})}^t = \dot{e} + \eta e, \quad \eta > 0
\]

where \( \eta \) is the convergence rate and any arbitrary positive constant as this guarantees the exponential decay of the error states. In order to achieve asymptotic convergence of the error state variables \( e(t) \) and \( \dot{e}(t) \) to zero, \( \lim t \to \infty e(t), \dot{e}(t) = 0 \), with a convergence rate \( \eta \), in the presence of a bounded uncertainty \( |A_{1,2,3,4}(X_1, X_2)| \leq \hat{A} \), the variable \( \sigma \) has to be driven to zero in a finite time. The quasi-sliding mode control law (26) can be used to drive \( \sigma \) to zero in a finite time:

\[
\omega_r = -\text{SAT}(\sigma, \varepsilon)
\]

\[
\text{SAT}(\sigma, \varepsilon) = -\Omega \frac{\sigma}{|\sigma|} + \varepsilon \quad \varepsilon \approx 0 \quad \varepsilon > 0
\]

where the sliding gain, \( \Omega \), can be calculated as below:

\[
\Omega = \hat{A} + \frac{\xi}{\sqrt{2}}
\]

The Sliding Mode Control (SMC) input, \( \omega_r \), in (26) is the rotational velocity of the shaft of \( M_2 \). We use an inner loop anti-windup proportional Integral (PI) controller (shown in Fig. 17) to control this velocity.

Where \( K_p, K_I \) and \( K_{aw} \) represent the proportional, integral and anti-windup gains, respectively; \( K_0 \) is a fixed conversation gain.

The role of the term \( \hat{A} \) in (27) is to compensate for the external bounded disturbance and any uncertainty of the system, whilst the term \( \xi/\sqrt{2} \) determines the reaching time to the sliding surface; choosing a larger value for \( \xi \) will lead to a shorter reaching time, \( T_s \). The sliding manifold reaching time can be calculated as:

\[
T_s \leq \frac{2\Lambda_g(0)}{\xi} = \frac{\sqrt{2} |\sigma(0)|}{\xi}
\]

where \( \Lambda_g \) is the Lyapunov candidate function for the explained SMC design. It is worth noting that from (26), we use the rotational velocity of the shaft of \( M_2 \) as the control input in our SMC.

Fig. 18 shows a schematic model of the designed hybrid PI-sliding mode velocity-force controller. Detailed information about the concept and design of the first-order sliding mode controller can be found in [53]-[55].

V. STABILITY AND ROBUSTNESS ANALYSIS

In this section we analysis the stability and robustness of the designed controller. To do so, from Fig. 16 and (24) we can write:

\[
e(t) = F_d - K_v \Delta L
\]

Substituting (29) into (25) we can write:

\[
\sigma_{(e, \dot{e})}^t = \dot{F}_d - K_v \Delta L + \eta \dot{F}_d - \eta K_v \Delta L
\]

and from (30) we can obtain:

\[
\sigma_{(e, \dot{e})}^t = \ddot{F}_d - K_v \Delta L + \eta \ddot{F}_d - \eta K_v \Delta L
\]

where for \( \eta K_v \Delta L \) we can write:

\[
\eta K_v \Delta L = \eta K_v \Delta L + \Delta L - \Delta L = \left( \eta K_v - 1 \right) \Delta L + \frac{\Delta L}{\eta}
\]

Substituting (32) into (31) we can get:

\[
\sigma_{(e, \dot{e})}^t = \frac{\ddot{F}_d - K_v \Delta L + \eta \ddot{F}_d - H \Delta L - \Delta L}{A(\Delta L, K_p \Delta L)}
\]
where \( \Delta L \) and \( \dot{\Delta L} \) are the speed and acceleration of the slider in the sliding system which are measurable using the encoder. The variable \( A(\Delta L, K_v, t) \) in (33) is called the system’s cumulative uncertainty-disturbance. We assume that this term is bounded, \( |A(\Delta L, K_v, t)| \leq \dot{A} \).

As explained above and from (24) and (25), driving the sliding variable to zero in finite time leads to an asymptotic convergence to zero on the error state variables \( e(t) \) and \( \dot{e}(t) \). In order to drive the sliding variable to zero, the controller should satisfy the following reachability condition [55]:

\[
\sigma \dot{\sigma} \leq -\bar{\xi}|\sigma|, \quad \text{where} \quad \bar{\xi} = \frac{\xi}{\sqrt{2}} \tag{34}
\]

where from (33) we can write:

\[
\sigma \dot{\sigma} = \sigma(A(\Delta L, K_v, t) - \Delta L) \leq |\sigma|\dot{A} - \sigma \Delta L \tag{35}
\]

and selecting:

\[
\dot{\Delta L} = \tilde{\Omega} \text{sign}(\sigma) \tag{36}
\]

by substituting (36) into (35) and from (34) we obtain:

\[
\sigma \dot{\sigma} \leq |\sigma|(\dot{A} - \tilde{\Omega}) = -\bar{\xi}|\sigma| \tag{37}
\]

From (37) we can conclude that the sliding mode gain should satisfy the reachability condition shown by (38) in order to guarantee the stability and robustness of the designed controller for a bounded disturbance-uncertainty, \( A(\Delta L, K_v, t) \).

\[
\tilde{\Omega} \geq \dot{A} + \bar{\xi} \tag{38}
\]

The term \( \dot{A} \) in (38) is used to overcome \( A(\Delta L, K_v, t) \), whilst the second term, \( \bar{\xi} \), determines the reaching time to the sliding surface; this reaching time can be calculated by substituting \( \bar{\xi} \) into (28). Any controller gain that satisfy condition (38) guarantees the stability of the designed controller.

V. CONTROL TEST PLATFORM

Fig. 19 shows the two-finger version of VSH1 as mounted on an ABB IRB 1200 with a 7 kg payload. We used this platform to test the controller. The grip force feedback is provided by a force sensor that was obtained from a 1D Force Sensing Resistor (FSR) mounted on the right fingertip. The cost of the sensor was £9. The main technical specifications of the sensor are reported in Table II.

![Image](image1.png)

FIGURE 19. The two-finger VSH1 mounted on the ABB robot.

The sensor (shown in Fig. 20) provides an analogue output as a variable resistance. We use a voltage divider circuit to transform the value of the resistance to a voltage value, which is readable by our control system. To emulate objects of different mechanical stiffnesses, \( K_v \), we designed a variable stiffness object (VSO), wherein springs with different stiffness can be exchanged.

![Image](image2.png)

FIGURE 20. Force Sensing Resistor used for the tests.

As shown in Fig. 21 the VSO consists of a spring in the centre which can be replaced to alter the stiffness of the object.

The VSO also consists of a linear potentiometer to measure the deformation of the VSO for our records. This data is then measured using an Arduino Mega, and sent via serial communication to a Windows PC using a baud rate of 9600 bps. We used four compression springs with different stiffness constants in our tests.

### Table II

<table>
<thead>
<tr>
<th>Specification of the Force Sensor Used in Our Hand.</th>
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<tbody>
<tr>
<td>Force sensitivity range</td>
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<tr>
<td>Pressure sensitivity range</td>
</tr>
<tr>
<td>Force resolution</td>
</tr>
<tr>
<td>Sensitivity to noise/vibration</td>
</tr>
</tbody>
</table>

![Image](image3.png)
Fig. 22 and 23 (a), (b), (c) and (d) depict the experimental results for the force control architecture explained by (26) and (27) and for the VSO with stiffnesses of $K_b = 1, 1.25, 2.6$ and $3.3 \text{ N/mm}$, respectively. To collect this data, the angle $θ$ was set to 0, 5, 10 and 15 degrees, respectively. The control gain and the error convergence rate in these tests were $Ω = 220 \text{ degree/sec}$ and $η = 20$. The dashed blue lines in subplots a, b, c and d in these figures depict the desired grip force, $F_d$, whereas the red curves show the measured force, $F_g$. As the figures show, thanks to the robustness of the designed SMC to overcoming uncertainty and environmental disturbances, the output of the controller always follows the desired input values with negligible overshoot and small steady-state errors. By comparing these subplots, it may also be noted that the steady-state errors decrease with increasing angle $θ$. This can be explained by the fact that the hysteresis band in VSH also decreases with increasing $θ$ (please recall Fig. 14). Finally, the error-based sliding variable $σ(e, ̇e)$ for these tests is shown in subplot (e) of the figures. As these subplots show, the sliding variable always remains at zero except at the moment when the desired grip force changes. This demonstrates the robustness of the designed controller in driving the error states to zero. At the instant that $F_d$ changes, the sliding variable jumps above or below zero for a very short period, then the controller drove it to zero. This proves the robustness of the controller in converging the error states to zero in a finite time.


**FIGURE 23.** Response of the hand controlled by the designed controller (solid red lines) to the step inputs with increasing amplitude (dashed blue lines) for (a) $\theta = 0^\circ$ and $K_b = 1$ (b) $\theta = 5^\circ$ and $K_b = 1.25$ (c) $\theta = 10^\circ$ and $K_b = 2.6$ (d) $\theta = 15^\circ$ and $K_b = 3.3$ N/mm (e) The sliding variable, $\sigma$, for this experiment.

Fig. 23 (a), (b), (c) and (d) depict the experimental results of the controller for two sinusoidal inputs and for the variable sets as $<K_b = 1\text{N/mm}, \theta = 0^\circ>$, $<K_b = 1.25\text{N/mm}, \theta = 5^\circ>$ , $<K_b = 2.6\text{N/mm}, \theta = 10^\circ>$ and $<K_b = 3.3\text{N/mm}, \theta = 15^\circ>$, respectively. In this figure, the dashed black and green curves depict the desired grip force, whereas the solid black and green curves show the measured grip force, $F_g$. As the figures show, the output of the controller always follows the desired sinusoidal inputs, with zero overshoot and small steady-state errors. Similar to the previous experiments, in this experiment the steady-state errors decrease with increasing angle $\theta$.

**FIGURE 24.** Response of the hand controlled by the designed controller (solid lines) to the sinusoidal inputs (dashed lines) for (a) $\theta = 0^\circ$ and $K_b = 1$ (b) $\theta = 5^\circ$ and $K_b = 1.25$ (c) $\theta = 10^\circ$ and $K_b = 2.6$ (d) $\theta = 15^\circ$ and $K_b = 3.3$ N/mm.

**VI. CONCLUSION**

A novel variable stiffness mechanism has been presented in this paper. The mechanism introduced provides a driving force for tendon-driven hands with an ability to control the position and stiffness of the fingers. The design consisted of two rotational servomotors. One of the servomotors, along with an integrated linear compression spring, was used to control the stiffness of the fingers whereas the other motor was responsible for changing the fingers’ positions. In order to control the apparent stiffness in the fingers, a mathematical model of the stiffness as a function of the shaft angle has been derived. Experimental results confirmed the effectiveness of the proposed variable stiffness mechanism. The hand design introduced is characterised by a large variability in stiffness, which is an essential requirement for a highly flexible handling system, and is particularly useful in food industry scenarios. The hand is also characterised by its fast response and small hysteresis band. The simplicity of its design besides providing a low-cost solution, guarantees the inherent reliability and robustness of this mechanism. The mechanism introduced can be used to control the grip force applied through simple control of the stiffness and compression of the integrated spring. Moreover, as explained, the integrated serial compliant element increases the robustness of the fixed gain controllers when dealing with objects of uncertain stiffness. In this paper we explained a PI-first order sliding mode velocity-force control architecture we designed to control the grip force by controlling the compression of the spring in the variable stiffness mechanism. We have shown experimentally, in the presence of unknown external disturbances and uncertainty of the model, that the designed SMC can robustly and in a finite time converge the error state variables to the origin and hence obtain the desired spring compression and, as a result, the desired grip force.
REFERENCES


